Paper C03
Fundamentals of Business Mathematics

This is the final article in the 'Back to basics' series that started in CIMA's student e-magazine, *Velocity*. It uses a case study to show how a range of quantitative techniques can be applied to solve a management problem
By Bob Scarlett

Our case-study business is called InmesCo, a mining firm based in the Republic of Gerania. It is considering how best to provide electricity to power one of its open-cast mining operations, which will be working at a remote location for 30 months.

The operation will have to generate its own electricity using facilities on site. Two alternative options have been identified for doing this:

- Rely wholly on a large diesel generator rented from a facilities company. The installation cost would be $35,000 and a rental fee of $10,000 would be payable on the last day of each quarter. It is forecast that the generator would use 10,000 litres of fuel every quarter.
- Acquire and install four wind turbines, which would have no residual value after 30 months, at a total cost of $300,000. They would provide all the electricity required on site for 80 per cent of the time, but small diesel generators already owned by InmesCo would have to provide the remainder. It is forecast that the small generators would use 2,000 litres of fuel every quarter.

InmesCo's current annual cost of money is 10 per cent.

The price of diesel is particularly volatile in Gerania. Based on recent experience, it is considered that the probabilities of different average prices throughout the 30 months are as follows:

- $2.00 per litre: 0.25 probability.
- $2.40 per litre: 0.30 probability.
- $2.80 per litre: 0.35 probability.
- $3.20 per litre: 0.10 probability.

The company has given us the task of evaluating the alternatives. In our analysis we will need to consider whatever issues are most relevant to basic project economics, InmesCo's cost of money and the sensitivity of the two options to fluctuations in the price of diesel and any other likely variables.

Specimen project analysis and report, with supporting narrative

The initial stage of the analysis is to consider a base case. The most obvious variable is the price of diesel and we can calculate an "expected price" as shown in table 1. The alternative prices weighted by their probabilities give an expected price per litre of $2.52, which is reasonable to use in a base-case analysis. For this we take cash flow figures from the information provided on costs and use the expected diesel price of $2.52 per litre as shown in tables 2 and 3. The discount rates we use are quarterly ones derived from InmesCo's 10 per cent annual cost of money. The relevant quarterly interest rate is calculated as follows: $\sqrt[4]{1.10} = 1.02411$. Therefore a 10 per cent annual interest rate corresponds to a 2.411 per cent quarterly rate. This gives

1. Expected price per litre of diesel

Price per l (A)	Probability (B)	A x B
$2.00	0.25	$0.50
$2.40	0.30	$0.72
$2.80	0.35	$0.98
$3.20	0.10	$0.32
		$2.52

2. Net present value of the costs of the diesel-only option

Quarter	Cash flow	Discount rate	Present value
0	$35,000	1.00000	$35,000
1	$10,000 + ($2.52 x 10,000) = $35,200	0.97645	$34,371
2	$35,200	0.95346	$33,562
3	$35,200	0.93101	$32,772
4	$35,200	0.90909	$32,000
5	$35,200	0.88769	$31,247
6	$35,200	0.86678	$30,511
7	$35,200	0.84637	$29,792
8	$35,200	0.82645	$29,091
9	$35,200	0.80699	$28,406
10	$35,200	0.78799	$27,737
			NPV $344,488

3. Net present value of the costs of the wind-turbines option

Quarter	Cash flow	Discount rate	Present value
0	$300,000	1.00000	$300,000
1	$2.52 x 2,000 = $5,040	0.97645	$4,921
2	$5,040	0.95346	$4,805
3	$5,040	0.93101	$4,692
4	$5,040	0.90909	$4,582
5	$5,040	0.88769	$4,474
6	$5,040	0.86678	$4,369
7	$5,040	0.84637	$4,266
8	$5,040	0.82645	$4,165
9	$5,040	0.80699	$4,067
10	$5,040	0.78799	$3,971
			NPV $344,313

4. Sensitivity analysis

Diesel price per l	Diesel-only option's NPV	Wind-turbines option's NPV
$2.00	$298,769	$335,169
$2.40	$333,938	$342,203
$2.80	$369,107	$349,237
$3.20	$404,276	$356,271

a Q1 discount rate of 1 ÷ 1.02411 = 0.997645, a Q2 discount rate of 1 ÷ 1.02411^2 = 0.95346 and so on.

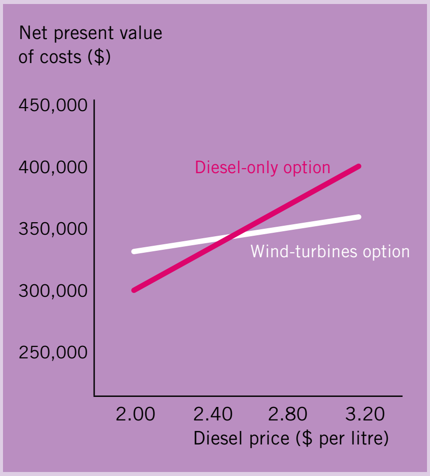
What can we conclude from our calculations so far? Not a lot. The analysis is inconclusive because it fails to show that either alternative offers a decisive advantage. The net present values (NPVs) of the costs associated with the two options are almost identical. So we have to look beyond the base case in order to decide which is the more favourable choice.

The main stated uncertainty is the price of diesel. We are told that this may vary between $2.00 and $3.20 on average over the 30 months, so let's perform a sensitivity analysis of the two options using the range of possible prices. The NPVs of the costs of the two options over the forecast range of diesel prices are shown in table 4, which are also represented in graphical form next to it.

Again, we can't conclude a great deal from the results of the sensitivity analysis. The diesel-only option is obviously more sensitive than the wind-turbines option to fuel price fluctuations, but not conclusively so. At both extremes of price, the difference in the NPVs of costs between the alternatives is little more than 10 per cent. If the price of fuel is low, this favours the diesel option and vice versa, but the advantages are not decisive either way.

In order to identify some significant distinguishing factor we will need to examine the economics of the venture. One feature of open-cast mining that differentiates it from other forms of mineral extraction is that it generally entails relatively low initial development costs. It does not involve sinking a shaft, building an offshore platform or constructing elaborate infrastructure. An operator merely moves excavators on to the site and starts digging, taking the minerals recovered to market using existing road and/or rail links. So an open-cast mining project, with few sunk

Analysis of the sensitivity of each option to fluctuations in the price of diesel



costs, can easily be abandoned. UK Coal's open-cast mining operation at Park Wall in Durham is a case in point. On 10 October the Weardale Railway website (www.weardale-railway.org.uk) reported that "the final train carrying coal from the Park Wall site departed on 2 October. These trains started in May 2011, usually running six times a week. The original plan was for them to run for 48 months, but operations at Park Wall stopped after 27 months. UK Coal decided that, given current market conditions, the limit of economic reserves at Park Wall had been reached."

So another key factor is our project's duration. Consider the possibility that the operation is terminated after only 18 months. The NPVs of the costs associated with the alternatives are shown in tables 5 and 6. This scenario gives the diesel-only option a crucial advantage: the NPV of the costs of the wind-turbine option is more than 40 per cent greater.

This reflects an appreciation of a key sensitivity inherent in the economics of the operation. Open-cast mining is a flexible activity, which favours options that entail low initial costs and easy exit routes. The diesel-only option would therefore be the better choice here.

It's clear that you can't always rely on a straight quantitative analysis of a base case or even on the sensitivity analysis of an obvious variable when you're assessing options. You need to consider the economics of each case and allow for the key variables that are characteristic of it.

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Further reading

B Scarlett, "Back to basics – risk and probability", Velocity, *August 2012 (bit.ly/CIMAvelocityRiskProbability).*

5. Net present value of the costs of the diesel-only option after 18 months

Quarter	Cash flow	Discount rate	Present value
0	$35,000	1.00000	$35,000
1	$10,000 + ($2.52 x 10,000) = $35,200	0.97645	$34,371
2	$35,200	0.95346	$33,562
3	$35,200	0.93101	$32,772
4	$35,200	0.90909	$32,000
5	$35,200	0.88769	$31,247
6	$35,200	0.86678	$30,511
		NPV	$229,462

6. Net present value of the costs of the wind-turbines option after 18 months

Quarter	Cash flow	Discount rate	Present value
0	$300,000	1.00000	$300,000
1	$2.52 x 2,000 = $5,040	0.97645	$4,921
2	$5,040	0.95346	$4,805
3	$5,040	0.93101	$4,692
4	$5,040	0.90909	$4,582
5	$5,040	0.88769	$4,474
6	$5,040	0.86678	$4,369
		NPV	$327,843