Article for Insider on Business Maths

Compound Interest

Compound interest is the very basis of financial mathematics and yet many students never really understand where the formula comes from. In the Business Mathematics *Study System* we develop the formula from first principles so that students will not only understand basic compounding but will later be better able to see how it is modified to deal with depreciation, equivalent rates of interest etc.

Suppose £*P* is invested at a fixed rate of interest of *r* per annum (where *r* is a proportion) and that interest is added at the end of each year; that is, it is compounded annually.

After one year, the value of the investment will be the initial investment £*P*, plus the interest accrued, £*rP*, and so will be

$$P + rP = P(1 + r)$$

During the second year, the interest accrued will be *r* times the amount at the end of the first year, and so will be *rP*(1 + *r*). The value at the end of the second year will be

$$P(1 + r) + rP(1 + r) = P(1 + r)(1 + r) = P(1 + r)^2$$

Proceeding in this way, after *n* years the value, £*V*, will be given by

$$V = P(1 + r)^n$$

This well-known formula is often referred to as the *compound interest* formula and is given in your exam.

As you will see, in financial mathematics we work with an *annual ratio* denoted by 1 + *r* rather than with the rate of interest.

Example:
An amount of £5,000 is invested at a fixed rate of 8 per cent per annum. What amount will be the value of the investment in five years' time, if the interest is compounded:
(a) annually?
(b) every six months?

Solution

(a) The only part of this type of calculation that needs particular care is that concerning the interest rate. The formula assumes that *r* is a *proportion*, and so, in this case:

r = 0.08

In addition, we have *P* = 5,000 and *n* = 5, so:

$V = P(1 + r)^5 = 5{,}000 \times (1 + 0.08)^5 = 5{,}000 \times 1.469328 = 7{,}346.64$

Thus the value of the investment will be £7,346.64.

(b) With slight modifications, the basic formula can be made to deal with compounding at intervals other than annually. Since the compounding is done at six-monthly intervals, 4 per cent (half of 8 per cent) will be added to the value on each occasion. Hence we use *r* = 0.04. Further, there will be ten additions of interest during the five years, and so *n* = 10. The formula now gives:

$V = P(1 + r)^{10} = 5{,}000 \times (1.04)^{10} = 7{,}401.22$

Thus the value in this instance will be £7,401.22.

In a case such as this, the 8 per cent is called a *nominal annual* rate, and we are actually referring to 4 per cent per six months.

In the *study system* we follow this theory with an exercise for students to practise the techniques of compound interest before moving on to slight modifications. We take the view in all the *study systems* that the best way to learn is by the dual approach of clear explanation and then lots of practise at solving problems for yourself.

Sandra Peers is the author of the CIMA *Study System* Business Mathematics.