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Management Accounting – Decision Management

The learning curve equation has a number of applications in the manufacturing sector. Fortunately, the formula itself is fairly straightforward to learn for paper P2.

The principle underlying learning curves is generally well understood: if we perform tasks of a repetitive nature, the time we take to complete subsequent tasks reduces until it can reduce no more. This is relevant to management accounting in the two key areas of cost estimation and standard costing.

Before we look at these we need to understand the maths. Imagine that we have collected the following information for the production of eight units of a product: it takes 1,000 hours to produce the first unit; 600 hours to produce the second unit; 960 hours to produce the third and fourth units; and 1,536 hours to produce the remaining four units. There is clearly a learning curve effect here, as the production time per unit is reducing from the initial 1,000 hours.

Learning curves are initially concerned with the relationship between cumulative quantities and cumulative average times (total cumulative time divided by cumulative quantity). The relationship in this case is shown in table 1. Notice that, as the cumulative quantity doubles, the cumulative average time reduces by 20 per cent. In other words, subsequent cumulative average times can be obtained by multiplying the previous cumulative average time by 80 per cent. This is an example of an 80 per cent learning curve. A learning curve is addressed in percentage terms, depending upon the relationship between the cumulative average times when the cumulative quantities are doubling. For example, if the cumulative average time were 1,000 hours at the production of the first unit, 700 hours at the production of the second, 490 hours at the fourth, 343 hours at the eighth and so on, this would be a 70 per cent learning curve.

The learning curve formula is needed when dealing with situations that do not fit into this doubling-up pattern.

A learning curve is geometric with the general form $Y = aX^b$.

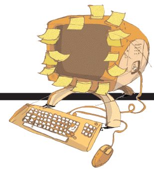
Y = cumulative average time per unit or batch.
a = time taken to produce initial quantity.
X = the cumulative units of production or, if in batches, the cumulative number of batches.
b = the learning index or coefficient, which is calculated as: log learning curve percentage ÷ log 2. So b for an 80 per cent curve would be log 0.8 ÷ log 2 = – 0.322.

Example one: unit accumulation

The first unit took 100 hours to produce. It is expected that an 80 per cent learning curve will apply. You are required to estimate the following times:

TABLE 1 CUMULATIVE QUANTITY VERSUS CUMULATIVE AVERAGE TIME

Cumulative quantity	Cumulative production time	Cumulative average production time per unit
1 unit	1,000 hours	1,000 hours
2 units	1,600 hours	800 hours
4 units	2,560 hours	640 hours
8 units	4,096 hours	512 hours



a The cumulative average time per unit to produce three units.
b The total time it will take to produce three units.
c The incremental time for the fourth and fifth units, in total.

The solutions are as follows:
a $Y = 100(3^{-0.322}) = 70.2$ hours per unit.
b We need to multiply Y by the cumulative number of units (X) to derive the total time: 70.2 x 3 = 210.6 hours.
c The incremental time to produce the fourth and fifth units equates to the total time to produce five units minus the total time to produce three units.
Cumulative average time to produce five units:
$Y = 100(5^{-0.322}) = 59.56$ hours per unit.
Total time for five units: 59.56 x 5 = 297.8 hours.
Incremental time: 297.8 – 210.6 = 87.2 hours.

Example two: batch accumulation
The first batch of a new product has just been made. The batch size was 20 units and the total time taken was 200 hours – ie, an average of 10 hours per unit. A 90 per cent learning curve is expected to apply. You are required to estimate the following:
a The cumulative average time for the first two batches.
b The total time to produce 40 units.
c The incremental time for 41 to 60 units – ie, a third batch of 20 units.

The solutions are as follows:
a This is a batch situation, so the Y value will be the cumulative average time per batch for two batches of 20 units.
a = 20 x 10 = 200 hours (ie, the time for the first batch).
b = log 0.9 ÷ log 2 = – 0.152.
X = 40 ÷ 20 = 2 (ie, two cumulative batches).
$Y = 200(2^{-0.152}) = 180$ hours per batch.
We could have avoided using the formula here and simply multiplied 200 by 0.9, because it was a doubling-up situation.
b A total time is needed so, as in example one, all we need to do is multiply Y by X:
2 batches x 180 hours per batch = 360 hours.
c The incremental time for the third batch equals the total time for 60 units minus the total time for 40 units.
We already have the total time for 40, so we need the total time for 60 (ie, X = 3):
$Y = 200(3^{-0.152}) = 169.24$ hours per batch.
Total time for 60 units = 169.24 x 3 = 507.72 hours.
Incremental time = 507.72 – 360 = 147.72 hours.

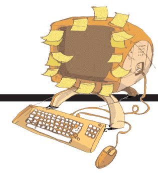
Now that we've looked at the mechanics, let's consider a couple of examples of how the learning curve can be applied.

Example three: cost estimation
BB plc uses a marginal costing system. You have been asked to provide calculations of total variable costs for a contract for one of its products, based on the following alternative situations:
1 A contract for one order of 600 units.
2 Contracts for a sequence of individual orders of 200, 100, 100 and 200 units. Four separate costings are required.

It's expected that the average unit variable cost data for an initial batch of 200 units will be as follows:
- Direct material: 15m^2 at £8 per m^2.
- Direct labour: department A: 8 hours at £8 per hour; and department B: 100 hours at £10 per hour.
- Variable overhead: 25 per cent of labour.

Labour times in department A are expected to follow an 80 per cent learning curve. Department B labour times are expected to follow a 70 per cent learning curve.

The cost estimates are as follows:
1 For one order for 600 units, we first need to define the appropriate values of a. We are dealing with a batch situation, so we need to define a as the time for the first batch of 200 units:
Department A: 200 x 8 = 1,600 hours.
Department B: 200 x 100 = 20,000 hours.
Next, we calculate the values of b:
Department A: log 0.8 ÷ log 2 = – 0.322.
Department B: log 0.7 ÷ log 2 = – 0.515.



The appropriate formulas are, therefore:
Department A: $Y = 1,600(X^{-0.322})$.
Department B: $Y = 20,000(X^{-0.515})$.
We can now proceed with the calculation of the total variable cost for a contract for 600 units:
Direct material: 600 x £120 per unit = £72,000.
Direct labour: we use the curve formulas to ascertain the labour times and then convert to cost. The cumulative quantity is 600 units, so X = 3 (ie, 600 ÷ 200).
Department A: $Y = 1,600(3^{-0.322}) = 1,123.28$ hours per batch.
This is the cumulative average time per batch for the cumulative three batches, so the total time is 1,123.28 x 3 = 3,370 hours.
The cost is, therefore, 3,370 x £8 = £26,960.
Department B: $Y = 20,000(3^{-0.515}) = 11,358.28$ hours per batch.
The total time is, therefore, 11,358.28 x 3 = 34,075 hours.
The cost is, therefore, 34,075 x 10 = £340,750.
Variable overhead: 0.25 x (26,960 + 340,750) = £91,928.
The total variable cost for the 600 units is as follows:

Direct materials:	£72,000
Direct labour:	£367,710
Variable overhead:	£91,928
Total:	£531,638

2 For sequential individual orders of 200, 100, 100 and 200 units, four separate costings are required. The costing for the first batch of 200 will be straightforward as we will simply use the average unit cost data initially given.
So the total variable cost for the first batch of 200 units is:

Direct material:	200 x 120	=	£24,000
Direct labour:			
Department A:	200 x 64	=	£12,800
Department B:	200 x 1,000	=	£200,000
Variable overhead:	0.25 x 212,800	=	£53,200
Total:			£290,000

For the next three batch orders we need to work out the incremental costs. Direct material cost is constant per unit, so it's not a problem. The variable overhead is 25 per cent of labour cost. Direct labour cost is affected by learning curves, so we need to calculate the incremental times using the learning curve formulas and then convert to cost (*see tables 2 and 3*). Then we can complete the costings for the second, third and fourth orders (*see table 4*).

Example four: standard costing
SC plc is establishing a revised standard cost for one of its products. The product was introduced at the start of 2004 when the standard variable cost for the first unit was as follows:
- Direct material: 10kg at £2 per kg.
- Direct labour: 10 hours at £8 per hour.
- Variable overhead: 10 hours at £4 per hour.
- Total variable cost per unit: £140.

TABLE 2 LEARNING CURVE FOR DEPARTMENT A
$Y = 1,600(X^{-0.322})$

Units	Cumulative units	X*	Y	Total time	Incremental time	Incremental cost at £8 per hour
200	200	1	1,600.0	1,600.0	1,600.0	12,800
100	300	1.5	1,404.2	2,106.3	506.3	4,050
100	400	2	1,280.0	2,560.0	453.7	3,630
200	600	3	1,123.3	3,369.9	809.9	6,479

* X = cumulative batches of the standard batch size of 200 (ie, the cumulative quantity ÷ 200).

TABLE 3 LEARNING CURVE FOR DEPARTMENT B
$Y = 20,000(X^{-0.515})$

Units	Cumulative units	X*	Y	Total time	Incremental time	Incremental cost at £10 per hour
200	200	1	20,000.0	20,000.0	20,000.0	200,000
100	300	1.5	16,230.9	24,346.4	4,346.4	43,464
100	400	2	14,000.0	28,000.0	3,653.6	36,536
200	600	3	11,358.3	34,074.9	6,074.9	60,749

* X = cumulative batches of the standard batch size of 200 (ie, the cumulative quantity ÷ 200).

TABLE 4 COSTINGS FOR THE SECOND, THIRD AND FOURTH BATCH ORDERS

Order	2nd (100 units)	3rd (100 units)	4th (200 units)
Direct material			
(£120 per unit)	12,000	12,000	24,000
Direct labour			
Department A	4,050	3,630	6,479
Department B	43,464	36,536	60,749
Variable overhead			
(25% of labour)	11,879	10,042	16,807
Total variable cost	**71,393**	**62,208**	**108,035**

During the year a 90 per cent learning curve was observed. The cumulative production at the end of the third quarter was 50 units. The budgeted production for the fourth quarter is 10 units.
You are required to calculate the following:
1 The standard cost per unit for the fourth quarter, assuming that the 90 per cent curve is still appropriate.
2 The standard cost per unit for the fourth quarter, assuming that peak efficiency was reached after the 50th unit was produced – ie, the learning curve had reached a steady state.